Is every abelian group A with $\mathrm{Ext}^1(A, \mathbb{Z}) = 0$ a free abelian group?

Suppose A is an abelian group such that every short exact sequence of abelian groups $0 \to \mathbb{Z} \to B \to A \to 0$ splits. Does it follow that A is free?

Tired of this problem?
https://goo.gl/xDC9Xu

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(True/False) Three clouds cover the plane, where a subset $A \subseteq \mathbb{R}^2$ is called a cloud around a if every line through a has a finite intersection with A.

The projective dimension of a module M is defined as the minimal length of a projective resolution of M. Let S be the ring $\mathbb{C}[x, y, z]$ and M be the S-module $\mathbb{C}(x, y, z)$. What is the projective dimension of M?

Tired of this problem?
http://goo.gl/iH3XVQ

Tired of this problem?
§4 of http://goo.gl/di2YrK

If X is a compact Hausdorff space, and f is an algebra homomorphism from $C(X)$ to some Banach Algebra, must f be continuous?

Let X be a set of reals such that for every sequence (ε_n) of positive reals, there exists a sequence of intervals (I_n) that cover X, with $|I_n| < \varepsilon_n$ for all n. Must X be countable?

Tired of this problem?
https://goo.gl/gTcFW8

Tired of this problem?
https://goo.gl/lzHhtM

(True/False) For every function f mapping $[0,1]$ into the set of countable subsets of $[0,1]$, there exist real numbers x and y such that $x \notin f(y)$ and $y \notin f(x)$.

(True/False) Define a matrix A_n to be a matrix of 0s and 1s with $A_n(i,j) = 1$ if $j = 1$ or $i|j$, 0 otherwise. For example, here is A_8:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then $\det(A_n) = O(n^{1/2+\epsilon})$ for every $\epsilon > 0$.

Tired of this problem?
https://goo.gl/I2kEDz

Tired of this problem?
http://goo.gl/FEvc2a

Is there an $n \geq 3$ such that

$$|\log(\mathrm{lcm}(1,2,\ldots,n)) - n| \geq \sqrt{n}\log^2(n)?$$

Here lcm denotes the least common multiple.

Let $\mathcal{N}_{(0,1)}$ be the space of functions

$$\mathcal{N}_{(0,1)} = \left\{ \sum_{k=1}^{n} c_k \rho\left(\frac{\theta_k}{x}\right),\ 0 < \theta_k < 1, \sum_{k=1}^{n} c_k = 0,\ n = 1,2,3,\ldots \right\}$$

where $\rho(u) = u - \lceil u \rceil$ is a fractional part of u. Is $\mathcal{N}_{(0,1)}$ dense in $L^2(0,1)$?

Tired of this problem?
4th comment in http://goo.gl/0sIZrW

Tired of this problem?
http://goo.gl/aMlPkr

(True/False) Up to order isomorphism, the real line is the only endless (no greatest or least element), dense, complete (nonempty bounded subsets have sup and inf) linear order in which every collection of disjoint intervals is countable.

(True/False) Given a Borel set in $\mathbb{R}^3$, project it in $\mathbb{R}^2$, take the complement and project it into $\mathbb{R}$. Then the resulting set is Lebesgue measurable.

Tired of this problem?
https://goo.gl/EJCgo1

Tired of this problem?
http://goo.gl/OFOuBW

(True/False) Let $H_n = \sum\limits_{j=1}^{n} \frac{1}{j}$. For each $n \geq 1$,

$$\sum_{d|n} d \leq H_n + \exp(H_n)\log(H_n),$$

with equality only for $n = 1$.

(True/False) Let

$$\mathrm{Li}(x) = \int_2^x \frac{dt}{\ln t}$$

and let $g(n)$ denote the maximal order of an element of the symmetric group S_n. Then for large enough n,

$$\log g(n) < \sqrt{\mathrm{Li}^{-1}(n)}$$

Tired of this problem?
http://goo.gl/f6cUCp

Tired of this problem?
http://goo.gl/ggwAz6

(True/False) Let

$$\Phi(u) = 2\sum_{n=1}^{\infty}(2n^4\pi^2 e^{\frac{9}{2}u} - 3n^2\pi e^{\frac{5}{2}u})e^{-n^2\pi e^{2u}}$$

Then

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi(\alpha)\Phi(\beta)e^{i(\alpha+\beta)x}e^{(\alpha-\beta)y}(\alpha-\beta)^2 \; d\alpha \; d\beta \geq 0.$$

Does the integral equation

$$\int_{-\infty}^{\infty} \frac{e^{-\sigma y}\phi(y)dy}{e^{e^{x-y}}+1} = 0$$

have a bounded solution $\phi(y)$ other than the trivial solution $\phi(y) = 0$, for some $\frac{1}{2} < \sigma < 1$?

Tired of this problem?
http://goo.gl/DhhGT3

Tired of this problem?
http://goo.gl/e60NEy